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LETTER TO THE EDITOR

Excitations in the integrable model with two- and three-particle interactions

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Abstract. Excitations in the integrable model with two- and three-particle interactions are calculated on the basis of the Bethe ansatz equations obtained in a previous paper. It is shown that the model exhibits two kinds of excitations, one connected with the massless particle–hole excitations, the other with massive excitations corresponding to the collective motion of the pseudoparticles.

Recently low-dimensional highly correlated systems have received a large amount of attention in connection with high-temperature superconductivity. A simple such model is the Hubbard model, which can be solved exactly in the one-dimensional case [1]. This model describes hopping of electrons on a one-dimensional chain with repulsion of two electrons on the same site due to Coulomb interaction. It can be presented as a two-sublattice model. In this case, electrons with different spins move along different sublattices and the four-fermion interaction between sublattices is the two-site one. This model was the single example of a discrete multi-sublattice integrable quantum system for a long time.

In previous paper [2] a new integrable model was proposed. To some extent it may be considered as an alternative to the one-dimensional Hubbard model. As in the Hubbard model, this model describes the motion of highly correlated electrons, but the four-fermion interaction between sublattices is a three-particle one. This leads to interesting physical properties. In particular, one can see the analogy between this model and the Anderson model of high-temperature superconductivity on the triangular lattice [3]. On the other hand the proposed model can be used in the study of quasi-one-dimensional conductors [4]. Therefore more detailed investigation of the properties of this model is desirable.

In a previous paper we derived a nested set of Bethe ansatz equations which determine the eigenstates and the corresponding energies and momenta. From these equations we calculated the ground state of the system. In this letter we calculate the excited states. We will use the ideas and methods which were developed for the Heisenberg chain [5] and the one-dimensional Hubbard model [4, 6–8].

The Hamiltonian of the model under consideration has the following form

$$H=\sum_{j=1}^{L}\sum_{\tau=1,2} c^{+}_{j(\tau)}c_{j+1(\tau)}+c^{+}_{j+1(\tau)}c_{j(\tau)} + U\sum_{j=1}^{L}\sum_{\tau=1,2}(c^{+}_{j(\tau)}c_{j+1(\tau)}+c^{+}_{j+1(\tau)}c_{j(\tau)})n_{j+\tau-1(\tau+1)} \qquad (1)$$

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where $c^+_{j(\tau)}$ ($c_{j(\tau)}$) creates (annihilates) an electron at the site j on the sublattice τ ($\tau = 1, 2$, $c_{j(3)} \equiv c_{j(1)}$) and $n_{j(\tau)} = c^+_{j(\tau)} c_{j(\tau)}$ is the corresponding number operator. As usual, periodic boundary conditions are imposed: $c_{L+1(\tau)} = c_{1(\tau)}$, where L is the number of sites. Thus the Hamiltonian (1) consists of a kinetic term describing the electronic motion between atomic sites of the same sublattice and the repulsion of two electrons on neighbouring sites of different sublattices due to the Coulomb interaction ($U > 0$). The model (1) can be presented in terms of spin operators [2].

In the previous work [2] we showed that the diagonalization of the Hamiltonian (1) can be reduced via a double Bethe ansatz to solving a set of coupled nonlinear equations. We also found the solution corresponding to the ground state of the system. The Bethe ansatz eigenstates of the Hamiltonian (1) are labelled by the sets of pseudo-momenta k_j and additional quantities Λ_β satisfying

$$
\begin{aligned}
&Lk_j + \sum_{\beta=1}^{m} \theta(k_j - \Lambda_\beta, \alpha') = 2\pi I_j \qquad (j = 1, 2, \ldots, n)\\
&\sum_{j=1}^{n} \theta(\Lambda_\beta - k_j, \alpha') - \sum_{\gamma=1}^{m} \theta(\Lambda_\beta - \Lambda_\gamma, 2\alpha') = 2\pi J_\beta \qquad (\beta = 1, 2, \ldots, m)\\
&\theta(k, \alpha') = 2\tan^{-1}[\coth(\alpha') \tanh \tfrac{1}{2}k] \qquad -\pi \leqslant \theta(k, \alpha) < \pi\\
&e^\alpha = (1 - v)^{-1} \qquad \alpha' = \tfrac{1}{2}\alpha
\end{aligned}
\tag{2}
$$

where I_j and J_β are integer (half-integer) numbers for even (odd) m and $n - m$, respectively. A solution of equations (2) corresponds to an eigenstate which is characterized by the total number of electrons n and the number of electrons on the first sublattice m. The energy and the momentum of this eigenstate are, respectively,

$$E = -2 \sum_{j=1}^{n} \cos k_j \tag{3}$$

$$P = \sum_{j=1}^{n} k_j = \frac{2\pi}{L}\left(\sum_{j=1}^{n} + \sum_{\beta=1}^{m} J_\beta\right). \tag{4}$$

The current into the system can be defined as in the one-dimensional Hubbard model [4]

$$j = \sum_{j=1}^{n} \sin k_j. \tag{5}$$

The ground state is characterized by the following values of I_j and J_β:

$$
\begin{aligned}
&I_j^0 = j - (n+1)/2 \qquad (j = 1, 2, \ldots, n)\\
&J_\beta^0 = \beta - (m+1)/2 \qquad (\beta = 1, 2, \ldots, m),\ m = n/2.
\end{aligned}
\tag{6}
$$

In the thermodynamic limit ($L \to \infty$, $n \to \infty$) the pseudo-momenta k_j tend to have a continuous distribution in the interval $[-Q, Q]$ with a continuous density $\rho(k)$ which is determined as a solution of integral equation [2]. The parameter Q is connected with the electron density $\rho = n/L$

$$\rho = \int_{-Q}^{Q} \rho(k)\, \mathrm{d}k. \tag{7}$$

In this letter we consider the excited states under the condition that the number of electrons n is conserved. These excitations consist of (i) massless excitations analogous to those of the Heisenberg chain and (ii) massive excitations which are connected with the collective motion of the pseudo-particles.

The first type of excitation corresponds to (*a*) 'holes' in the k-distribution

$$I_j = I_j^0 + \theta_{jj0} \qquad J_\beta = J_\beta^0$$
$$\theta_{jj0} = \begin{cases} 0 & j < j_0 \\ 1 & j \geq j_0 \end{cases} \tag{8}$$

and (*b*) 'particles' in the k-distribution

$$I_j = I_j^0 + A\delta_{nj} \qquad J_\beta = J_\beta^0. \tag{9}$$

The unification of these excitations corresponds to annihilation of a particle with momentum k_h and creation of a particle with momentum k_p. Thus we have an excitation of particle–hole type.

The second type of excitation corresponds to (*a*) 'holes' in the Λ-distribution

$$J_\beta = J_\beta^0 + \theta_{\beta\beta 0} \qquad I_j = I_j^0 \tag{10}$$

and (*b*) 'strings' in the Λ-distribution. Strings are families of complex Λ which, for L tending to infinity, have the same real part. For an l-string, these are located at

$$\Lambda_\beta^{l,j} = \Lambda_\beta^l + \mathrm{i}(l+1-2j)\alpha \qquad j = 1, 2, \ldots, l. \tag{11}$$

In calculating the excitations we follow the methods given by previous authors [4, 5] who studied the one-dimensional Heisenberg and Hubbard models. We write equations (2) with numbers I_j and J_β which are given by (8) and subtract from them the corresponding equations for the ground state. A straightforward manipulation leads to the energy of the excitations of the first type in the thermodyanmic limit

$$E - E_0 = 2\left[\int_{-Q}^{Q} \tilde{\rho}(k) \sin k \, \mathrm{d}k + \cos k_h - \cos k_p\right]. \tag{12}$$

Here E_0 is the ground-state energy for an infinite system [2]. The corresponding momentum and current are

$$P = \int_{-Q}^{Q} \tilde{\rho}(k) \, \mathrm{d}k + k_p - k_h$$
$$j = 2\left[\int_{-Q}^{Q} \tilde{\rho}(k) \cos k \, \mathrm{d}k + \sin k_p - \sin k_h\right]. \tag{13}$$

In these equations the function $\tilde{\rho}(k)$ is determined as a solution of the integral equation

$$2\pi\tilde{\rho}(k) - \int_{-Q}^{Q} \varphi(k-k')\tilde{\rho}(k') \, \mathrm{d}k' = \int_{k_h}^{k_p} \varphi(k-k') \, \mathrm{d}k' \tag{14}$$

where

$$\varphi(k) = \frac{1}{2} \sum_{n=-\infty}^{\infty} \frac{\mathrm{e}^{-|n|\alpha}}{\cosh(n\alpha)} \mathrm{e}^{\mathrm{i}nk}.$$

Equations (12)–(14) give the two-parameter excitations. If one parameter is fixed we obtain either hole-excitation ($k_p = Q$) or particle-excitation ($k_h = Q$). The solution of the integral equation (14) can be obtained with the help of numerical calculation or using perturbation theory. In particular in the strong-coupling limit ($\alpha \to \infty$) we have

$$E - E_0 = 2(\cos k_h - \cos k_p)\left[1 + \frac{e^{-2\alpha}}{\pi}(Q - \tfrac{1}{2}\sin 2Q)\right] + O(e^{-4\alpha})$$

$$P = \frac{2\pi}{2\pi - Q}(k_p - k_h) + \frac{4\,e^{-2\alpha}\sin Q}{2\pi - Q}$$

$$\times\left[\sin k_p - \sin k_h + \frac{\sin Q}{2\pi - Q}(k_p - k_h)\right] + O(e^{-4\alpha})$$

$$j = 2\left[\sin Q\left(\frac{k_p - k_h}{2\pi - Q}\right) + \sin k_p - \sin k_h\right]$$

$$+\left[1 + \frac{e^{-2\alpha}}{\pi}\left(\frac{2\sin^2 Q}{2\pi - Q} + Q + \tfrac{1}{2}\sin 2Q\right)\right] + O(e^{-4\alpha})$$

$$Q = \pi\rho(1 + \tfrac{1}{2}\rho)^{-1} - 2\sin^2[\pi\rho(1 + \tfrac{1}{2}\rho)^{-1}]\,e^{-2\alpha} + O(e^{-4\alpha}).$$

Consider now the excitations of the second type. The number of holes in the Λ-distribution is always even. In the simplest case this number equals two. This state is obtained as a result of transfer of one electron from one sublattice to another. For such a transfer it is necessary to get over the electron repulsion. As a result there is a gap in the spectrum of excitations corresponding to holes in the Λ-distribution. In order to calculate the value of this gap consider equations (2) for values of I_j and J_β which are given by (6) but at $m = (n/2) - 1$, i.e. we suppose that the holes in the sequence of $J(\Lambda_\beta)$ are located at

$$J_{\beta_1} = -\frac{n}{4} \qquad J_{\beta_2} = \frac{n}{4}. \tag{15}$$

Following the previous work [2] we obtain for the energy of this state

$$E - E_0 = 2\Delta \qquad \Delta = \int_{-Q}^{Q} \rho_1(k)\cos k\,dk \tag{16}$$

where $\rho_1(k)$ is the solution of the integral equation

$$2\pi\rho_1(k) - \int_{-Q}^{Q} \varphi(k - k')\rho_1(k')\,dk' = 2\Phi_2(k + \pi)$$

$$\Phi_2(k) = \frac{1}{2}\sum_{n=-\infty}^{\infty} \frac{e^{ink}}{\cosh(n\alpha)}. \tag{17}$$

At $\alpha \to \infty$ we have the following expansion for the gap

$$\Delta = \frac{\sin Q}{2\pi - Q} - \frac{e^{-\alpha}}{\pi}\left[\frac{2\sin^2 Q}{2\pi - Q} + Q + \tfrac{1}{2}\sin 2Q\right] + O(e^{-2\alpha}).$$

Consider now the case of an arbitrary location of holes in the Λ-distribution. To calculate the energy and momentum in this case one has to write equations (2) for numbers I_j and J_β (10) and subtract from them the corresponding equations for the choice (15). Further calculation, which is analogous to the previous consideration, leads to the following results

$$E-E_0=2\left[\Delta+\int_{-Q}^{Q}\tilde{\varphi}(k)\sin k\,\mathrm{d}k\right]$$
$$P=\int_{-Q}^{Q}\tilde{\varphi}(k)\,\mathrm{d}k \qquad j=2\int_{-Q}^{A}\tilde{\rho}(k)\cos k\,\mathrm{d}k. \tag{18}$$

The function $\tilde{\rho}(k)$ is a solution of the integral equation

$$2\pi\tilde{\rho}(k)-\int_{-Q}^{Q}\varphi(k-k')\tilde{\rho}(k')\,\mathrm{d}k'=-\int_{-\pi}^{\Lambda_1}\Phi_2(k-\Lambda)\,\mathrm{d}\Lambda+\int_{\Lambda_2}^{\pi}\Phi_2(k-\Lambda)\,\mathrm{d}\Lambda. \tag{19}$$

At $\alpha\to\infty$ we have

$$E-E_0=\sum_{i=1}^{2}\varepsilon(\Lambda_i) \qquad P=\sum_{i=1}^{2}p(\Lambda_i) \qquad j=\sum_{i=1}^{2}j(\Lambda_i)$$

$$\varepsilon(\Lambda)=\Delta+\frac{2\mathrm{e}^{-\alpha}}{\pi}(Q-\tfrac{1}{2}\sin 2Q)(1+\cos\Lambda)+\mathrm{O}(\mathrm{e}^{-2\alpha})$$

$$P(\Lambda)=\frac{Q}{2\pi-Q}\Lambda+\mathrm{e}^{-\alpha}\sin Q\sin\Lambda+O(\mathrm{e}^{-2\alpha})$$

$$J(\Lambda)=\frac{2\sin Q}{2\pi-Q}\Lambda+\frac{\mathrm{e}^{-\alpha}}{\pi}\left(\frac{2\sin^2 Q}{2\pi-Q}+Q+\tfrac{1}{2}\sin 2Q\right)\sin\Lambda+\mathrm{O}(\mathrm{e}^{-2\alpha}).$$

In addition to holes, excited states may contain conjugate pairs of complex Λ_β (10). These excitations, though not contributing to the energy and momentum, do play a role in the classification of states leading to the degeneracy of the energy levels.

Thus the energy spectrum of the model under consideration exhibits two kinds of excitations. The first kind is connected with massless particle-hole excitations and the second one with massive excitations corresponding to collective motion of the pseudoparticles.

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